

# Secular Decrease in the Inclination of Artificial Satellites

RAJENDRA C. NIGAM\*  
*Smithsonian Astrophysical Observatory,  
 Cambridge, Mass.*

MERSON and King-Hele<sup>1</sup> noticed a marked decrease in the inclination of Sputnik II and suggested, among the possible causes, the rotation of the atmosphere. Wildhack<sup>2</sup> studied the effect of the transverse atmospheric drag on the inclination, and showed a secular decrease caused by atmospheric rotation. In view of the smallness of this effect, he was skeptical that it could be used to obtain any definite information on winds and tides in the upper atmosphere. Sterne,<sup>3</sup> however, from his analysis of the inclination of Sputnik II, suggested the probability of an atmospheric wind blowing from west to east at about 130 mph at heights between 150 and 250 km. The aim of this paper is to extend this study by using the increased accuracy in the orbital elements which has become available in the past four years.

For verification of the theory, the authors just cited have confined themselves to the orbital data on Sputnik II given by Cornford.<sup>4</sup> In this paper the author has used orbital data on 1958 Alpha (Explorer I), 1958 Gamma (Explorer III), and 1958 δ2 (Sputnik III). Furthermore, this paper simultaneously uses two different equations derived independently to compute the secular decrease in the inclination for each of the three satellites. The first equation gives the rate of change of inclination with period:

$$\frac{di}{dP} = \frac{\omega_s}{12\pi} \sin i (1 - e)^2 \left( \frac{1 - e}{1 + e} \right)^{1/2} \cdot \frac{k}{1 + d} \cdot \frac{1 + (1/8c)\{1 - 4ek + 4e[(3 + 2e)/(1 - e^2)]\} + \cos 2\omega[1 - (1/8c)\{15 + 4ek + 4e[(5 + 6e)/(1 - e^2)]]\}}{1 + (1/8c)\{1 - 8ek - [4e^2/(1 - e^2)]\}} \quad (1)$$

where  $c = (ae/H) > 2$ , and no assumption has been made about the smallness of  $e$ . Equation (1) differs from Sterne's<sup>3</sup> Eq. (16) in that before the square-root sign this paper has  $(1 - e)^2$  instead of  $(1 - e)$ . The symbols used here are the same as Sterne's.

Equation (2) represents the change of inclination with respect to eccentricity,

$$\frac{di}{de} = \frac{\omega_s}{4n} \sin i \frac{(1 - e)^{3/2}}{(1 + e)^{1/2}} \cdot \frac{k}{1 + d} \cdot \frac{1 + (1/8c)\{1 - 4ek + 4e[(3 + 2e)/(1 - e^2)]\} + \cos 2\omega[1 - (1/8c)\{15 + 4ek + 4e[(5 + 6e)/(1 - e^2)]]\}}{1 - (1/8c)\{3 + 4ek + [4e^2/(1 - e^2)] + [4edk/(1 + d)]\}} \quad (2)$$

where again,  $c > 2$ , and no restriction has been imposed on  $e$ .

The effect of the atmospheric rotation on orbital inclination has been studied by several other authors, including Bosanquet,<sup>5</sup> Plimmer,<sup>6</sup> Cook and Plimmer,<sup>7</sup> Cook,<sup>8</sup> and Vinti.<sup>9</sup> The treatments of all except Sterne suffer from the drawback that their equations are correct to the first power of eccentricity only.

That the two results derived from Eqs. (1) and (2) are almost the same for the satellites under consideration, as Table 1 shows, doubly confirms the validity of this procedure.

### Comparison of Observation with Theory

Equations (1) and (2) are strictly valid only if there were no long-periodic perturbations owing to the odd harmonics

Received March 14, 1963. This work was supported in part by NASA Grant Nsg 87-60. The assistance of Maria C. Gutierrez in computations is acknowledged gratefully.

\* Astronomer, Research and Analysis Division; now with ITT Intelcom Inc., Bailey's Cross Roads, Va.

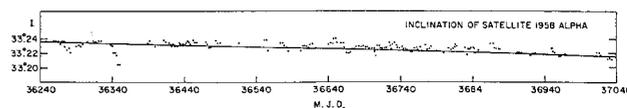


Fig. 1 The solid line shows the least-squares linear expression for inclination against the observed values with the effect of the third harmonic removed

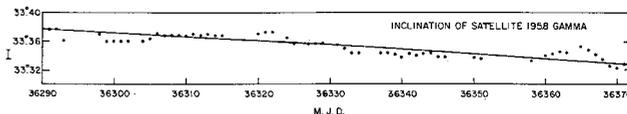


Fig. 2 The solid line shows the least-squares linear expression for inclination against the observed values

in the earth's potential. It is therefore necessary that one subtracts the long-periodic perturbations owing to the odd harmonics from the orbital elements before using these equations. Neglect to do this can cause significant error. The lunisolar and radiation-pressure perturbations, though important, are insignificant for the satellites considered here.

### Satellite 1958 Alpha

Orbits spread over 800 days from February 14, 1958 through April 24, 1960 were used. The values of inclination are plotted in Fig. 1. Using Kadakia's least-squares program, the following expression for the inclination was obtained:

$$I = (33^\circ.2254 \pm 16) - (0^\circ.261 \pm 45) \times 10^{-4} [t - \text{March 21, 1959}] \pm 0^\circ.0003 \text{ [standard deviation (S.D.)]}$$

### Satellite 1958 Gamma

Orbits spread over an interval of 80 days from March 29, 1958 to June 17, 1958 were used. The values of inclination are plotted in Fig. 2. The least-squares expression for the inclination is

$$I = (33^\circ.3798 \pm 22) - (0^\circ.655 \pm 52) \times 10^{-3} [t - \text{March 29, 1958}] \pm 0^\circ.0007 \text{ (S.D.)}$$

### Satellite 1958 Delta 2

All reliable orbits spread over an interval of 419 days from December 7, 1958 to January 30, 1960 were used. Special precaution had to be taken in analyzing the data on this satellite because of the near-critical inclination of its orbit. The amplitude of the long-periodic perturbation on the perigee distance is 1 km, and that on the inclination is 0°.0002. This, however, enabled one to ignore the long-periodic perturbations. The author was rather fortunate in having for this satellite two sets of photoreduced observa-

**Table 1 Derived wind velocity from secular decrease in inclination**

1) Satellite	2) Average perigee height, km	3) $(\Delta i)_1$	4) $(\Delta i)_2$	5) $(\Delta i)_{theo}$	6) $(\Delta i)_{obs}$	7) Residuals	8) Wind velocity (implied), mph
1958 Alpha	355	-0°.0152	-0°.0152	-0°.0152	-0°.0208	-0°.0056 ± 25	368 ± 164
1958 Gamma	188	-0°.0452	-0°.0454	-0°.0453	-0°.0524	-0°.0071 ± 42	157 ± 93
1958 Delta 2	221	-0°.0448	-0°.0453	-0°.0451	-0°.0535	-0°.0084 ± 34	186 ± 75

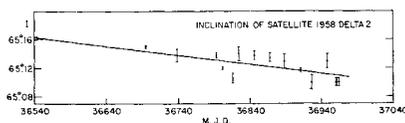
tions, separated by more than 400 days. The inclinations used here are plotted, along with their standard errors, in Fig. 3. The least-squares expression for the inclination is

$$I = (65°.1629 \pm 03) - (0°.1277 \pm 82) \times 10^{-3} [t - \text{December 7, 1958}] \pm 0°.0003 \text{ (S.D.)}$$

**Results**

Table 1 gives the values of the secular decrease in the inclinations of the three satellites. Column 3 shows the theoretical decrease in inclination derived by using Eq. (1), and column 4 shows the same decrease obtained through Eq. (2). Column 5 gives the mean of columns 3 and 4. Column 6 gives the observed secular decrease computed from least-squares expressions for inclination. Column 7 lists the residuals, along with their standard deviations.

These results are more accurate than others so far published. For each one of the three satellites, the author clearly has a higher secular decrease in the inclination than that predicted on the assumption of the solid-body rotation of the atmosphere, which conforms to the results of other authors. Column 8 of Table 1 gives the velocity of the wind, necessary to explain the higher observed secular decrease in the inclination of the three satellites. The results appear to suggest winds moving at high speeds in the upper atmosphere. If winds moving at speeds high enough to explain the higher



**Fig. 3 The solid line shows the least-squares linear expression for inclination against the observed values with their standard deviations**

secular decrease in inclination seem unlikely, one should obviously look for some other phenomenon that might account for the increased transverse acceleration on the motion of the artificial satellites.

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**Graphical Method for Optimization of Cesium-Surface Ionizer Materials**

G. KUSKEVICS\*

*Electro-Optical Systems, Inc., Pasadena, Calif.*

A graphical method is proposed for the optimization of cesium ionizer material, structure, and operating point. Proper selection of the ionizer material, its structure and work function, and the design point would optimize the propellant-utilization efficiency, power efficiency, and, to some extent, life of the cesium-ion engine. The method is illustrated 1) by optimization of the design point for a solid tungsten ionizer using exponential and linearized plots and 2) by selection of optimum material based on preliminary experimental critical temperature  $T_c$  curves and theoretical neutral fraction  $\alpha$  curves.

**1. Introduction**

THE performance of a surface-ionization cesium ionizer is determined by ionization efficiency, ion-generation energy efficiency, and lifetime. The ionization efficiency  $\beta$  is defined as the ratio of the ion efflux to the total cesium efflux. The ion-generation energy efficiency  $\eta$  is the ratio of the ionization potential to the ion-generation energy. These ionizer figures of merit directly determine the propellant-utilization efficiency and the overall ion-engine power efficiency, respectively. Historically, ionizers and their design points have been selected to give very high (near unity) ionization efficiency regardless of the ion-generation energy efficiency. The best overall performance is obtained if these two opposing figures of merit are simultaneously optimized. The ionizer lifetime sets an upper limit of operating temperature. Similarly, limits are set by breakdown voltage on the ion-current density and by electrode life on the ionization efficiency.

**2. Graphical Optimization of the Ionizer Design Point**

A graphical method for selection of the optimum design-operating point for a given ionizer is illustrated by its application to a solid tungsten ionizer. The ionization efficiency  $\beta$  vs  $T_c$  and the specific ion-generation energy,  $p/j$  vs  $T_c$  plots are superimposed in Fig. 1. The  $\beta$  curve is a plot of the Saha-Langmuir equation for a constant work function

Received by ARS July 29, 1962; revision received April 15, 1963.

\* Senior Scientist, Ion Physics Department. Member AIAA.